Prove $\dfrac{\cot x - \tan x}{\cos x + \sin x} = \dfrac{\cos x - \sin x}{\cos x \sin x}$.

$$= \frac{\dfrac{\cos x}{\sin x} - \dfrac{\sin x}{\cos x}}{\cos x + \sin x} \cdot \frac{\cos x \sin x}{\cos x \sin x}$$

$$= \frac{\cos^2 x - \sin^2 x}{(\cos x + \sin x)\cos x \sin x}$$

$$= \frac{\cancel{(\cos x + \sin x)}(\cos x - \sin x)}{\cancel{(\cos x + \sin x)}\cos x \sin x}$$

$$= \frac{\cos x - \sin x}{\cos x \sin x} \qquad \boxed{\text{QED}}$$

$$\frac{\tan^2 t - \sec^2 t}{1 - \cos^2 t}$$

$$= \frac{-1}{\sin^2 t}$$

$$= -\csc^2 t$$

$$\frac{\cos(-\beta) - \sec\beta}{\csc\beta + \sin(-\beta)}$$

$$= \frac{\cos\beta - \sec\beta}{\csc\beta - \sin\beta}$$

$$= \frac{\cos\beta - \frac{1}{\cos\beta}}{\frac{1}{\sin\beta} - \sin\beta}$$

$$= \frac{\frac{\cos^2\beta - 1}{\cos\beta}}{\frac{1-\sin^2\beta}{\sin\beta}}$$

$$= \frac{\frac{-\sin^2\beta}{\cos\beta}}{\frac{\cos^2\beta}{\sin\beta}}$$

$$= \frac{-\sin^2\beta}{\cos\beta} \cdot \frac{\sin\beta}{\cos^2\beta}$$

$$= -\frac{\sin^3\beta}{\cos^3\beta}$$

$$= -\tan^3\beta$$

$$\frac{1+\cos\theta}{\tan\theta+\sin\theta}=\cot\theta$$

$$= \frac{1+\cos\theta}{\frac{\sin\theta}{\cos\theta}+\sin\theta}\cdot\frac{\cos\theta}{\cos\theta}$$

$$= \frac{\cos\theta(1+\cos\theta)}{\sin\theta+\sin\theta\cos\theta}$$

$$= \frac{\cos\theta\,\cancel{(1+\cos\theta)}}{\sin\theta\,\cancel{(1+\cos\theta)}}$$

$$= \frac{\cos\theta}{\sin\theta}$$

$$= \cot\theta \qquad \boxed{\text{QED}}$$

$$\cot^4 y + \csc^2 y = \csc^4 y - \cot^2 y$$

$$= (\csc^2 y - 1)^2 + \csc^2 y$$

$$= \csc^4 y - 2\csc^2 y + 1 + \csc^2 y$$

$$= \csc^4 y - \csc^2 y + 1$$

$$= \csc^4 y - (\csc^2 y - 1)$$

$$= \csc^4 y - \cot^2 y \quad \boxed{\text{QED}}$$